$$\frac{R}{n}+\frac{r}{a/n}, \quad \frac{aR+rn^2}{an}.$$

Placing the first differential coefficient of this expression equal to zero we have

$$\frac{2an^2rdn-a^2Rdn-an^2rdn}{a^2n^2}=0.$$

From which

$$rn^2=aR, \quad n^2=\frac{aR}{r}.$$

Replacing the value of a/m for one factor in n^2,

$$n\frac{a}{m}=\frac{aR}{r}, \quad \frac{n}{m}=\frac{R}{r}, \quad R=\frac{nr}{m}.$$

Or the external resistance equals the total internal resistance. This is seen to be a minimum value for the expression differentiated since the value of the second differential coefficient is greater than zero for the positive value of n, —the only value it can have.

THE RADIUS OF THE TERRESTRIAL SPHEROID.

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If there be nothing *new* under the sun, it may not be uninteresting to expand the *old*.

Represent the earth's equatorial radius by a, the geographical latitude by ϕ, and the geocentric latitude by ϕ'; then since $x^2/a^2+y^2/b^2=1$, we have $\tan\phi=-dx/dy$, and $\tan\phi'=y/x$. Also, since $b^2=a^2(1-e^2)$, we have

$$y^2=a^2(1-e^2)-(1-e^2)x^2 \text{ and } y/x=(1-e^2)\tan\phi.$$

$$\therefore\ x=\frac{a\cos\phi}{\surd(1-e^2\sin^2\phi)} \text{ and } y=\frac{a(1-e^2)\sin\phi}{\surd(1-e^2\sin^2\phi)} \ldots \ (1).$$

Now, the radius of the terrestrial spheroid for any latitude ϕ, is $\rho=\surd(x^2+y^2)$.

$$\therefore\ \rho=a\sqrt{\left(1-\frac{e^2(1-e^2)\sin^2\phi}{1-e^2\sin^2\phi}\right)},=a\surd[1-e^2(1-e^2)(\sin^2\phi+e^2\sin^4\phi)].$$

By assuming $e^2=1-f^2$ and $\sin^2\phi=\frac{1}{2}(1-\cos2\phi)$, Encke obtains the series

$$\log\rho = 9.9992747 + 0.0007271\cos 2\phi - 0.0000018\cos 4\phi,$$

in which the equatorial radius is unity.

Making $x=\rho\cos\phi'$ and $y=\rho\sin\phi'$, then the former equation of (1) may be written $e^2=\dfrac{\rho^2\cos^2\phi'-a^2\cos^2\phi}{\rho^2\sin^2\phi\cos^2\phi'}$; and by means of this value of e^2, the elimination of e^2 from the latter equation of (1) may be effected.

$$\therefore\ \rho^2\sin^2\phi' = \frac{(\rho^2\sin^2\phi\cos^2\phi' - \rho^2\cos^2\phi' + a^2\cos^2\phi)^2}{\rho^2\sin^2\phi\cos^2\phi\cos^2\phi'}.$$

$$\therefore\ \rho = a\sqrt{\left(\frac{\cos\phi}{\cos\phi'\cos(\phi-\phi')}\right)}\ldots.(a).$$

Formula (a) may be deduced in another way; by assuming that

$$e\sin\phi = \sin\psi\ldots.(2),$$

we obtain

$$\rho\sin\phi' = a(1-e^2)\sin\phi\sec\psi\ldots(\alpha),\text{ and }\rho\cos\phi' = a\cos\phi\sec\psi\ldots.(\beta).$$

From (α) and (β) by easy deductions,

$$\rho\sin(\phi-\phi') = \tfrac{1}{2}ae^2\sin 2\phi\sec\psi\ldots.(\alpha'),\text{ and }\rho\cos(\phi-\phi') = a\cos\psi\ldots.(\beta').$$

From (2) and (β') we have, respectively,

$$e=\sin\psi/\sin\phi\text{ and }\cos\psi=\rho\cos(\phi-\phi')/a\,;$$

and after transforming (α'), etc., we obtain

$$\rho\sin(\phi-\phi'), = a\times\frac{\cos\phi}{\sin\phi}\times\frac{1-\cos^2\psi}{\cos\psi}, = a\times\frac{\cos\phi}{\sin\phi}\times\frac{a^2-\rho^2\cos^2(\phi-\phi')}{a\rho\cos(\phi-\phi')}.$$

$$\therefore\ \rho^2 = \frac{a^2\cos\phi}{\sin(\phi-\phi')\cos(\phi-\phi')\sin\phi + \cos^2(\phi-\phi')\cos\phi}.$$

Expanding this denominator, combining terms, etc., we have (a) by a second method of reduction.

In order to obtain formula (a) by a third method, we remember that $a^2/b^2 = \tan\phi/\tan\phi'$ and that $x^2+(a^2/b^2)y^2=a^2$; or after obvious transformations,

$$\rho^2\cos^2\phi' + (a^2/b^2)\rho^2\sin^2\phi' = a^2,\ \text{ or }\ \left[\cos^2\phi' + \left(\frac{\sin\phi\times\cos\phi'}{\cos\phi\times\sin\phi'}\right)\sin^2\phi'\right]\rho^2 = a^2,$$

from which formula (a) is readily deduced.